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First-Order Modal Predicate Logic, Modal Realism and the Problem of Transworld Identity

Bachelor Thesis

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1 Introduction

1.1 The Concept of Modality

Modality is a fairly old concept and in common language a widely used tool, reaching as far back as to Aristotle and allowing imaginative sentences like "If my mother had married another man, I would be another person". While classical logic only cares about what is the case, modals are able to encapsulate apodictic and problematic sentences, concepts which are of great interest not only to philosophy. Besides logical space, which regards every contingent sentence as possible and only tautologies as necessary, we can discuss temporal logic, where we talk about things that could be or could have been, or doxastic logic, which is all about the things one should believe or is able to believe, to name some but not all logics from the modal family. Generally we can differentiate between modal truths of sentences, which we call truths *de dicto*, and the modal truths of objects, which is called truths *de re*.

But what actually is meant by such modal sentences, is still not so clear as one might imagine. Only in the latter half of the last century did modality evolve into a thorough logical concept by adding operators like $\Box$, $\Diamond$ or @ (for *actually*) to classical logic and especially the many world semantics brought the concept of modality to another level. But still there is no clear consensus about how to interpret these semantics, for example what possible worlds are and whether they actually exist.

Here I will sketch the technical basics of first-order modal predicate logic, trace the genuine realist interpretation from David Lewis and its criticisms. Furthermore, I will outline the problem of transworld identity and discuss its solutions.

1.2 Why Modal Logic Is Important

Modals cannot be expressed by classical logic because every sentence in the form of "If A would have been true, B would be the case" could only be translated as $A \supset B$, and so would be automatically true if A were not the case or if B were the case. One can also see that we cannot express modality by adding operators behaving classical logical. For

example: You cannot no find a unary connective # with whom $\#A \supset A$ is true for any formula A and for any tautology B the sentence $\#B$ is true, which are both desired traits for a necessity operator. So, there is really no way around extending classical logic when you want to talk about modals in logic.

So, one could easily formulate true conditional formulas, where the conditional is possible but actually false and where the consequence could be everything imaginable. That could be something like "If the surface temperature of the earth were 100 °C, there would be talking donkeys", which is of course pure nonsense and something we do not want from a logic handling modality, but it would be true under classical logic.

Of course, one could believe in the *categorical hypothesis*, which roughly states that a theory which only describes all things in our universe would already be a complete theory of the world.[1] This would discard the concept of modality as a whole and would not allow any meaningful thoughts like "x has the ability to do y". Indeed, it seems such a theory may encapsulate a pure determinism, because it could not even consider sentences like "If I push down the vase, it would break", because whether the vase is truly breakable or not, the sentence would come out as true as long as I do not push down the vase, if we only consider factuals and classical logic. In a sense we will come back to this discussion later, when talking about the existence of possible worlds and whether they exist in some form, which is in my opinion the same question as whether modality is really a sensible concept or whether we should accept the categorical hypothesis or at least a version of it.

2 First Order Modal Predicate Logic

2.1 The Formulas

First order classical logic is one of the most well understood tools in logic and mathematics, not only because of its simplicity but also because of its role as foundation of mathematics and other disciplines. This is why a good first step for building a modal lan-

[1] See Joseph Melia. *Modality*. Central Problems of Philosophy. Taylor & Francis, 2014, p. 1.

guage is to begin with fist-order predicate logic and extend it with modal operators.[2] So our language $\mathscr{L}$ has the following symbols:

(variables)	$x, y, z, ...x_1, x_2, ...$
(predicate letters)	$P_1^1, P_2^1, P_3^1, ...P_1^2, P_2^2, ...P_1^3, ...$ (where P_m^n is an n-place relation symbol)
(connectives)	$\wedge, \vee, \supset, \equiv, \neg$
(quantifiers)	$\forall, \exists$
(modal operators)	$\Box, \Diamond$

and to prevent any ambiguity, we have also brackets ( ) helping us out. One could also consider a functionally complete language with more or fewer symbols, but I will include all these, as they represent our natural language in a very handy way. Additionally, we could add individual constants and functions to our language, but to keep it a little simpler, especially when considering truth evaluation, we will work only with valuations. So, sentences from $\mathscr{L}$ can be defined inductively as follows.

At first, an *atomic formula* is an expression of the form $P(x_1, x_2, ..., x_n)$, where P is an n-place predicate letter and $(x_1, x_2, ...x_n)$ is an ordered set of individual variables. All variables occurring in an atomic formula are called free. Furthermore: If A and B are *formulas*, so are

$\neg A$	(with all free variables in A being free in $\neg A$)
$(A \circ B)$	(with $\circ$ as any two place connective and with all free variables occurring in A and B being free in $(A \circ B)$)
$\Box A$, $\Diamond A$	(with all free variables in A being free in $\Box A$ and $\Diamond A$)
$(\forall x)A$, $(\exists x)A$	(with all free variables in A being free in $(\forall x)A$ and $(\exists x)A$, except for x itself, which is called bound)

As it is common, I may sometimes leave out brackets when writing down formulas, as long as there are no ambiguities. Formulas, where every variable is bound, are called *sentences*. On their own only sentences can be evaluated because a free variable does not point to any specific object or individual and therefore it cannot be said anything about it. So, to be able to evaluate formulas with free variables, we need a valuation v that maps every variable x

[2]This chapter will be mainly based on the book from Melvin Fitting and Richard L. Mendelsohn. "First-Order Modal Logic". In: 1998, especially from chapter four *Quantified Modal Logic*, pp. 81-115.

to an individual $v(x)$ from our domain $\mathcal{D}$. Two valuations v and w are called *variant with respect to a variable x* (x-variant) only if all mappings are identical, except for possibly the valuation of x. Or in short: v is x-variant to w, iff $v(y) = w(y)$ for all variables y, except for possibly x.

2.2 Possible Worlds Semantics

But how do you evaluate modal formulas? In normal classical predicate logic our models would be made up by a set of individuals (a domain) including their respective properties and relations, which we can denote as sets of objects and individuals or sets of n-tuples of objects and individuals, to which these properties and relations apply. But now we run into the complication that we want to evaluate actually non-existent properties and relations of things i.e. counterfactuals. And we would also consider things and their properties, which in turn may not even exist themselves. So, where do we find them in our model, when we cannot find them in our domain? The answer usually given since their establishment in the 1960s is *possible worlds*. What these things actually are is up to debate for later, but we will just call them that without any real meaning put onto them, even when we might interpret them later as instances of different times or ways the world could have been. We usually imagine our own world as one of many possible worlds in our semantics, even though it might not be crystal clear what *our world* really is. So, in that sense modal logic is not an alternative, but an addition to classical logic, because in our special possible world, we call our own, we get exactly these sentences as true which also are under classical logic true.

Between the possible worlds we need some kind of accessibility relation, which determines which worlds are modally accessible from one another. It would be a strange approach to simply assume any modality so that everything that is true in some world is possible in our world, so we have to restrict this reach of possibility. That means by definition that something is only possible in a world Γ, if it is true in a world Δ which is accessible from Γ. This of course depends also on the modal context that is considered. For example, it is nomologically possible, so under the laws of nature in our own world, that earth does not have a moon. While it may be nomologically impossible that something

goes faster than the speed of light, even though it may be logically possible.

2.2.1 Varying Domain Semantics

Now we should consider one important aspect of modality and that is that there are not only possible properties of things but also possible things themselves. For example, unicorns and talking donkeys may be possible and may therefore reside in a possible world, but they do not exist in our own world. And vice versa: It may be possible for something, which exists in our world, to not exist. We could work our way around this problem, by adding a predicate for existing **E**, which may even be acceptable, if one believes in all possible things being in every world at once. But now one must specify in modal language whether *there is* something or *there exists* something. If one would agree that all possible things (all possibly existing things) have the *property* of being, the existence quantifier becomes somewhat a *being quantifier* and saying something exists would be stated as $(\exists x)(\mathbf{E}x)$.

But a somewhat more sophisticated approach would be to take the discussion of actually non-existing things seriously and to work with varying domain semantics. Thus, we need to work with a domain function instead of an unchanging domain.

2.2.2 Modal Models

Now we have everything we need to build a model, that can evaluate first-order modal predicate formulas. A model $\mathcal{M} = \langle \mathcal{G}, \mathcal{R}, \mathcal{D}, \ell \rangle$ consist of a set of possible worlds $\mathcal{G}$, a set of accessibility relations between these worlds $\mathcal{R}$, a domain function $\mathcal{D}$, which gives us a set of objects and individuals depending on the world $\Gamma \in \mathcal{G}$, and an interpretation ℓ, which assigns to each n-place predicate symbol P from our language and each world Γ a set of n-tuples of individuals from $\mathcal{D}(\Gamma)$. We can imagine ℓ as a function which gives us for all ordered pairs (P,Γ), where P is any predicate symbol with n places and Γ is any world, a set of n-tuples of individuals from $\mathcal{D}(\Gamma)$.

When we only talk about the worlds, their relation between them and their domains, we talk about a frame $\mathcal{F} = \langle \mathcal{G}, \mathcal{R}, \mathcal{D} \rangle$. Generally these objects do not see, so important, because most properties of frames can also be directly applied to models, but they are

useful when talking about formulas that can come true already by the properties of $\mathcal{R}$ and $\mathcal{D}$, as we will see later.

Now we can define the truth of a formula A in a model $\mathcal{M} = \langle \mathcal{G}, \mathcal{R}, \mathcal{D}, \ell \rangle$ in a world Γ under a valuation v (written as $\mathcal{M}, \Gamma \Vdash_v A$) inductively as follows.

$\mathcal{M}, \Gamma \Vdash_v P(x_1, x_2, ..., x_n) \iff (v(x_1), v(x_2), ...v(x_n)) \in \ell(P, \Gamma)$

$\mathcal{M}, \Gamma \Vdash_v \neg A \iff \mathcal{M}, \Gamma \nVdash_v A$

$\mathcal{M}, \Gamma \Vdash_v A \wedge B \iff \mathcal{M}, \Gamma \Vdash_v A$ and $M, \Gamma \Vdash_v B$

$\mathcal{M}, \Gamma \Vdash_v A \vee B \iff \mathcal{M}, \Gamma \Vdash_v A$ or $\mathcal{M}, \Gamma \Vdash_v B$ or both

$\mathcal{M}, \Gamma \Vdash_v A \supset B \iff \mathcal{M}, \Gamma \nVdash_v A$ or $\mathcal{M}, \Gamma \Vdash_v B$ or both

$\mathcal{M}, \Gamma \Vdash_v A \equiv B \iff$ either $\mathcal{M}, \Gamma \Vdash_v A$ and $\mathcal{M}, \Gamma \Vdash_v B$ or $\mathcal{M}, \Gamma \nVdash_v A$ and $\mathcal{M}, \Gamma \nVdash_v B$

$\mathcal{M}, \Gamma \Vdash_v \Diamond A \iff$ for some Δ with $\Gamma \mathcal{R} \Delta : \mathcal{M}, \Delta \Vdash_v A$

$\mathcal{M}, \Gamma \Vdash_v \Box A \iff$ for every Δ with $\Gamma \mathcal{R} \Delta : \mathcal{M}, \Delta \Vdash_v A$

$\mathcal{M}, \Gamma \Vdash_v \exists x A \iff$ for some x-variant w of v in $\mathcal{D}(\Gamma) : \mathcal{M}, \Gamma \Vdash_v A$

$\mathcal{M}, \Gamma \Vdash_v \forall x A \iff$ for all x-variant w of v in $\mathcal{D}(\Gamma) : \mathcal{M}, \Gamma \Vdash_v A$

The formulation in the last two rows "in $\mathcal{D}(\Gamma)$" is because we must have a valuation that maps at least onto the domain of the world, otherwise it may map a free variable to something outside of $\mathcal{D}(\Gamma)$. While it makes sense to talk about truth under a valuation when regarding to formulas, one can see that it does not matter what valuation I choose for a sentence, because it would only change the meaning of free variables, which do not exist in formulas. So, we can abbreviate truth of sentences to be only in a model and a world because the truth value does not depend on our valuation ($\mathcal{M}, \Gamma \Vdash A$). And additionally, we can call sentences, which are true in all worlds of a model, true in a model ($\mathcal{M} \Vdash A$).

For an easy example, we may take a model $\mathcal{M} = \langle \mathcal{G}, \mathcal{R}, \mathcal{D}, \ell \rangle$ with $\mathcal{G} = \{\Gamma, \Delta\}$, $\mathcal{R} = \{(\Gamma, \Delta)\}$, $\mathcal{D}(\Gamma) = \mathcal{D}(\Delta) = \{c\}$ and ℓ is defined by $\ell(P^1, \Gamma) = \{c\}$ and $\ell(P^1, \Delta) = \emptyset$. Note that c is not considered a part of our language we use, because it is in our domain, which includes the objects themselves (I could have also chosen to put a *more real* object into $\mathcal{D}$ like a table or a donkey). So, saying that c has the property P in our model must be stated by saying that $P(x)$ is true in our model in world Γ under any valuation v with $v(x) = c$ or in short $\mathcal{M}, \Gamma \Vdash_v P(x)$ with $v(x) = c$. This would be a true formula in that case. Additionally

we could state that it is possible for c to not be P, written as $\mathcal{M}, \Gamma \Vdash_v (P(x) \wedge \Diamond \neg P(x))$ with $v(x) = c$, because in Δ which is accessible from Γ we have $\mathcal{M}, \Delta \Vdash_v \neg P(x)$ with $v(x) = c$. And if one wants to make a sentence out of it, we can say for example that there exists an object in the domain instead of directly referring to c: $\mathcal{M}, \Gamma \Vdash \exists x(P(x) \wedge \Diamond \neg P(x))$.[3]

An interesting fact I would like to point out is that we now always refer to objects and their properties in respect to a world. Which in some way makes sense, because for example when we refer to an object or an individual over a period of time, we may make statements like "He was married in 1950 but was unmarried in 1873"[4]. We do not say that the person we refer to is both married and unmarried at the same time, but the property of being married is linked to a temporal constraint. If we were to make an easier statement without any temporal restriction about the person, the truth of it would depend only on the question if he is currently married. Or to be clearer: The sentence "He is married" is only true if he is indeed married at the moment this sentence is said. This corresponds to the fact that we, living in our own world, can make direct true statement about what is true in our world, but need a modal signifier when regarding to something that is true in another world.

2.3 Identity and Existence

The relation of two things being identical is a very distinct one compared to other relations like, for example, something being bigger or taller than something else. For when I make a claim like "x is identical to y" I mean to say that both variables x and y refer to one and the same object. We would thus say that a formula $(x = y)$ is true in a model $\mathcal{M}$ in a world Γ under a valuation v iff both x and y get mapped onto the same object from our domain $v(x) = v(y) \in \mathcal{D}(\Gamma)$ (or in short $\mathcal{M}, \Gamma \Vdash_v x = y \iff v(x) = v(y) \in \mathcal{D}(\Gamma)$). Additionally to being a relation that every object has to itself and only to itself, $=^2$ must be an equivalence

[3] Because we have a sentence we may be inclined not to specify under which valuation we consider it, even though it is never denoted that way in Fitting and Mendelsohn, "First-Order Modal Logic".

[4] Penelope Mackie and Mark Jago. "Transworld Identity". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Winter 2017. Metaphysics Research Lab, Stanford University, 2017.

relation, which means that it is transitive, reflexive and symmetric.[5] [6]

We already talked about the property of existing, which is often written as $\mathbf{E}!$, but we dismissed it in favour of introducing varying domain semantics. But with the introduction of identity, we are able to define existence in a varying domain model by stating that there is an individual identical to the individual we want to regard to. So, an equivalent formulation of $\mathbf{E}!(x)$ would be $(\exists y)(x = y)$.

2.4 Different Modal Logics

We already touched on the notion of applying modal logic to different concepts, which gives us for example doxastic, temporal and logical space itself. So, depending on our modal context and thus the way we like to understand our modal operators, they must obey certain rules. For example, when considering modal logic, we would like a rule that gives us for every tautology A the formula $\Box A$ as true, because logical truths should be true in all worlds. When adding the axiom of distribution $\Box(A \supset B) \supset (\Box A \supset \Box B)$, we get the basic modal Kripke logic **K**. [7] But in general, we find some formulas we might want to come out as true but are not **K**-valid, like $\Box A \supset A$. And so, by adding more and more axioms, we can build stronger and stronger modal logics up to for example **S5**.

The weaker modal logic **S4** can be built from **K** by adding the following two axiom schemes.

$$\text{(T)} \qquad \Box A \supset A$$

$$\text{(4)} \qquad \Box A \supset \Box\Box A$$

Note that A can be any formula here, even with free variables. When we now also add the axiom

$$\text{(5)} \qquad \Diamond A \supset \Box\Diamond A$$

[5] See Harold Noonan and Ben Curtis. "Identity". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Fall 2022. Metaphysics Research Lab, Stanford University, 2022.

[6] Models that have identity behaving in that way are also called *normal models*, but because we will always assume normality as here described, I will just call them models as well.

[7] Depending on what proof system one is using, one could also define a modal logic by adding rules instead of axioms.

we get the modal logic **S5**. When we consider possible worlds semantics with frames again, we may, instead of adding these axioms, express the different modal logics by demanding different properties from the accessibility relation $\mathcal{R}$ in our frame $\mathcal{F}$. One can proof that the axioms are always true in a model, iff $\mathcal{R}$ has the following properties respectively:

(T) Reflexive $\quad \forall \Gamma \in \mathcal{G} : \mathcal{R}(\Gamma, \Gamma)$

(4) Transitive $\quad \forall \Gamma, \Delta, \Omega \in \mathcal{G} : (\mathcal{R}(\Gamma, \Delta) \wedge \mathcal{R}(\Delta, \Omega)) \supset \mathcal{R}(\Gamma, \Omega)$

(5) Euclidean $\quad \forall \Gamma, \Delta, \Omega \in \mathcal{G} : (\mathcal{R}(\Gamma, \Delta) \wedge \mathcal{R}(\Gamma, \Omega)) \supset \mathcal{R}(\Delta, \Omega)$

We might also say that a *frame* is reflexive, transitive or euclidean iff its accessibility relation has this property. This viewpoint of frames with different properties is helpful to imagine the meaning of modality, even if it might be ontologically quite converse.

We will now show that the frame property of transitivity is equivalent to (4) being an axiom in every model based on such a frame. We will first show that if we have a model $\mathcal{M}$ of a transitive frame, (4) will always come out true.

So, suppose for some world $\Gamma \in \mathcal{G}$ and any valuation v we have $\mathcal{M}, \Gamma \Vdash_v \Box A$. Therefore for every world Δ with $\mathcal{R}(\Gamma, \Delta)$ it is the case that $\mathcal{M}, \Delta \Vdash_v A$. If we were not to find any such Δ, $\Box\Box A$ must also come out as true, because $\Box A$ would be true in every world which is accessible from Γ, because there are no such worlds. If there is no world Ω with $\mathcal{R}(\Delta, \Omega)$, we would again get $\mathcal{M}, \Gamma \Vdash_v \Box\Box A$, because A would be true in every accessible world from Δ, because there are no such worlds. To complete our case discrimination, we assume there is an Ω with $\mathcal{R}(\Delta, \Omega)$. Due to transitivity we must also have $\mathcal{R}(\Gamma, \Omega)$, so because of our assumption we get $\mathcal{M}, \Omega \Vdash_v A$. And because Ω can be arbitrary we have $\mathcal{M}, \Delta \Vdash_v \Box A$. And due to the arbitrariness of Δ we have $\mathcal{M}, \Gamma \Vdash_v \Box\Box A$. Now, we can apply the theorem of deduction to get $\mathcal{M}, \Gamma \Vdash_v \Box A \supset \Box\Box A$, which makes $\Box A \supset \Box\Box A$ a theorem for every model based on a transitive frame and every formula A, because $\mathcal{M}$, Γ, v and A can be arbitrarily chosen, except for the transitivity of the frame.

We will show the other direction by contraposition. So we suppose a model $\mathcal{M} = \langle \mathcal{G}, \mathcal{R}, \mathcal{D}, \ell \rangle$ of a non-transitive frame $\mathcal{F}$ with $\mathcal{G} = \{\Gamma, \Delta, \Omega\}$, $\mathcal{R} = \{(\Gamma, \Delta), (\Delta, \Omega)\}$, $\mathcal{D} = \{c\}$ and ℓ can be completely defined by $\ell(\Gamma, P^1) = \ell(\Omega, P^1) = \emptyset$ and $\ell(\Delta, P^1) = \{c\}$. Therefore, we get $\mathcal{M}, \Delta \Vdash_v P(x)$ for a valuation v with $v(x) = c$. This in turn gives us $\mathcal{M}, \Gamma \Vdash_v \Box P(x)$.

On the other hand, we have $\mathcal{M}, \Omega \nVdash_v P(x)$ (v being the same as before), which gives us $\mathcal{M}, \Delta \nVdash_v \Box P(x)$, which in turn gives us $\mathcal{M}, \Gamma \nVdash_v \Box\Box P(x)$. Altogether, this shows per definition that $\mathcal{M}, \Gamma \nVdash_v \Box\Box P(x) \supset \Box P(x)$. Therefore, we have shown that there is at least one case of a non-transitive frame, where $\Box\Box A \supset \Box A$ is not true in a world for a formula A, which is the contrapositive to the fact that if $\Box\Box A \supset \Box A$ is always true in a model for any formula A, the frame the model is based on must be transitive.

■

2.5 Barcan and Converse Barcan Formulas

While introducing possible worlds semantics, we also touched upon the concept of models and frames with constant domains and varying domains. We could also say that constant domain models are actually a subclass of varying domain models with the condition $\forall \Gamma \in \mathcal{G} : \mathcal{D}(\Gamma) = \mathcal{D}(\mathcal{M}) := \bigcup_{\Gamma \in \mathcal{G}} \mathcal{D}(\Gamma)$. This gives us a certain behaviour of our modal operators, like the ability to change in some cases the order between them and quantifiers, because quantifiers will always quantify over the whole domain of any world. For example, if $\Diamond(\exists x)P(x)$ is true, we will always find an object in some accessible world from ours with the property P. But because of the constant domain, this object must also exist in our original world, so we could also find in this world an object, which has the property P in some accessible world, which gives us $(\exists x)\Diamond P(x)$. One can see that this would also work in the other direction and with the permutation of $\Box$ and $(\forall x)$.

A weaker version of this would be to only allow certain permutations of modal operators and quantifiers. We can express this through the Barcan and the converse Barcan scheme. Which are defined like this

Barcan formula:	$(\forall x)\Box\Phi \supset \Box(\forall x)\Phi$
	$\Diamond(\exists x)\Phi \supset (\exists x)\Diamond\Phi$
converse Barcan formula:	$\Box(\forall x)\Phi \supset (\forall x)\Box\Phi$
	$(\exists x)\Diamond\Phi \supset \Diamond(\exists x)\Phi$

Note that Φ may be any of our formulas, so even though we may talk about the (converse) Barcan formula, we actually mean all Barcan formulas, who follow this scheme. Thus,

we have two different versions of each scheme. We say that *the* Barcan formula is true in a frame iff all Barcan formulas are true in all models based on that frame. That means that no matter what formula we put for Φ and what our interpretation is, the constituting Barcan formula is always true. And conversely, if we could find a formula Φ, so that the Barcan formula comes out as false, we say that the Barcan formula is not true in the frame. The same goes for the converse Barcan formula.

Similar to our different modal logics we can see how the truth of these formulas correspond to possible worlds semantics. The properties we need to introduce for frames and models are monotonicity and anti-monotonicity. These are defined like this: If for every $\Gamma, \Delta \in \mathcal{G}$ with $\mathcal{R}(\Gamma, \Delta)$ it is that $\mathcal{D}(\Gamma) \subseteq \mathcal{D}(\Delta)$, then we call the frame *monotonic*. And conversely, if for every $\Gamma, \Delta \in \mathcal{G}$ with $\mathcal{R}(\Gamma, \Delta)$ it is that $\mathcal{D}(\Delta) \subseteq \mathcal{D}(\Gamma)$, then we call the frame *anti-monotonic*. So, while in a monotonic frame, we never lose any individuals when going to another accessible world, so the domain can only get bigger. In an anti-monotonic frame the domain from one world to an accessible one can only become smaller, so we never get any new individuals.

And now we can combine these properties of frames with the (converse) Barcan formula. One can proof the following two equivalences.

1. The converse Barcan formula is always true in $\mathcal{M} \Longleftrightarrow \mathcal{M}$ is monotonic
2. The Barcan formula is always true in $\mathcal{M} \Longleftrightarrow \mathcal{M}$ is anti-monotonic

So not only does the Barcan and converse Barcan formula allow us to permute quantifiers with modal operators in a certain way, but they also tell us something about the existence of things throughout possible worlds. Thus, we can also reformulate the (converse) Barcan formula using the property of existence $\mathbf{E}$, we could define through identity. So just like monotonicity say that if something exists in a world, it must also exist in any world accessible from it, we can state that it must exist necessarily. So, we get the following as equivalent in a frame $\mathcal{F}$:

1. $\mathcal{F}$ is monotonic.
2. The converse Barcan formula is true in every model based on $\mathcal{F}$.
3. $\mathcal{M} \Vdash (\forall x)(\mathbf{E}(x) \supset \Box \mathbf{E}(x))$ for every $\mathcal{M}$ based on $\mathcal{F}$.

4. $\mathcal{M} \Vdash (\forall x)\Box\mathbf{E}(x)$ for every $\mathcal{M}$ based on $\mathcal{F}$.

And in the same sense, if we are in an anti-monotonic frame, we can express the anti-monotonicity by stating that if something exists possibly in a world, it must exist in that same world. Here we get the following equivalences for a frame $\mathcal{F}$:

1. $\mathcal{F}$ is anti-monotonic.

2. The Barcan formula is true in every model based on $\mathcal{F}$.

3. $\mathcal{M} \Vdash (\forall x)(\Diamond\mathbf{E}(x) \supset \mathbf{E}(x))$ for every $\mathcal{M}$ based on $\mathcal{F}$.

Interestingly and maybe even unfortunately, we cannot find a compact formula here, like $(\forall x)\Box\mathbf{E}(x)$ for monotonicity, where we do not rely on additional connectives. But if we were to add an *actually operator* @, which is redundant when considering specific Kripke models[8], we could reformulate $(\forall x)(\Diamond\mathbf{E}(x) \supset \mathbf{E}(x))$ into $\Box(\forall x)@\mathbf{E}(x)$.

If we have a frame which is both monotonic and anti-monotonic we call its domain *locally constant*, as it behaves in many ways just like a constant domain frame. Indeed it is even the case that a sentence is only true in all constant domain models iff it is also true in all locally constant domain models.[9] This seems obvious, considering that a fundamental property of both is that one can permute $\Box$ with $(\forall x)$ and $\Diamond$ with $(\exists x)$ as one pleases. The only real difference is that in a locally constant domain frame there may be patches of worlds which have no accessibility relation between them and may have therefore differing domains in their worlds.

Of course, all the talk about monotonicity and anti-monotonicity has philosophically quite some consequences, considering for example that it states that everything, that does actually exist, exists necessarily, or that all possible things do in fact actually already exist. Sometimes, one may want to have monotony or anti-monotony in a certain context, for example in epistemic logic, we would not want to lose individuals we actually know about. But generally, the Barcan formula or the converse Barcan formula may not be seen as true.

[8] See Allen Hazen, Benjamin Rin, and Kai Wehmeier. "Actuality in Propositional Modal Logic". In: *Studia Logica* 101 (June 2013).

[9] See Fitting and Mendelsohn, "First-Order Modal Logic", p. 113.

3 Modal Realism

3.1 Lewis's Thesis about Possible Worlds

For the moment we have just discussed the technical details of modality, while we only touched on the metaphysical notions of it. The metaphysics of modality is still highly controversial in some areas, because there are many different theses concerning the realness and constituency of possible worlds, like Quinian Scepticism, Modalism[10] or ersatzism[11]. *Modal realism*, also called genuine realism or sometimes even extreme realism, is another important thesis on interpreting modality in natural language. Most famously it was put forward by David Lewis. There are of course other variations of modal realism, but we will focus on the one that Lewis had in mind.

His thesis is that our physical world we live in is only one of many and that these possible worlds are concrete objects.[12] But what does that actually mean? Lewis imagines our whole cosmos throughout time and space as one complete world. The so called worldmate relation is defined by Lewis as all things that have a spatiotemporal distance.[13] Therefore all things that bear this relation to one another are in fact in a world. And here we come already to an interesting conclusion, because it directly rules out the possibility of there being a thing in two different worlds at once, because the relation of having spatiotemporal distance is not only reflexive and symmetric but in fact also transitive. This will be important later, when talking about transworld identity. Another way of differentiating the possible worlds is by saying that they have no causal relation between them.[14]

The worlds themselves underlie the notion that they all together constitute all things that are possible,[15] so when there might have been no moon, there is at least one world where earth has no moon, which seems in some sense already a little bit circular. But for now, we just take Lewis for his word and imagine that these worlds just exist on their own, and everything we may say that is not logically false is the case in some possible

[10] See Melia, *Modality*.

[11] See David Lewis. *On the Plurality of Worlds*. Wiley-Blackwell, 1986, p. 136.

[12] See ibid., p. 2.

[13] See ibid., pp. 70-71.

[14] See ibid., p. 78.

[15] See ibid., p. 2.

world. This notion allows him to talk also about so called alien properties, which cannot in any way be described by the properties we have here in our world.[16] So one may not be able to talk concretely about a possible world, because we would not have the vocabulary describing its alien properties. There are nevertheless some ways to describe worlds with non-alien properties, for example by using the *principle of recombination*. This states that two non-worldmates are able to coexist in another third world by there being duplicates of them.[17] I may even formulate it a little bit simpler: we can have a thing in a world by it being a duplicate of something else existing in another world. Lewis uses the term *duplicate* to mean things that share the same intrinsic properties.[18]

Another point I like to mention, even though it seems Lewis does not value it too much, is the point of *concreteness*. He argues that his worlds do not only have the property of being, but also, they are concrete objects. Still he admits that not all parts of concrete worlds are concrete and "it also seems that to say that is to say something very ambiguous indeed."[19]

This all together is of course a very unintuitive claim, to think that there are, besides our own, other concrete worlds which are spatiotemporally and causally isolated from ours. While Lewis does see the controversy in his statement, he thinks there are good reasons to believe in a plurality of worlds. On the one hand he argues for possible worlds as a useful concept, because it would allow us to quantify over more than there actually is and therefore allow statements about *possibilia*,[20] i.e. things that do exist in some possible worlds and therefore have the property of being. On the other hand, one can define properties as the set of all this- and otherworldly things that have these properties, just like we did with our interpretation of predicate symbols in quantified modal logic. And one can constitute a proposition by the set of worlds where it is true.[21] But the important part is that "Lewis attempts to reduce modality by pairing modal claims with proposed analyses

[16] See Lewis, *On the Plurality of Worlds*, p. 91.

[17] See ibid., p. 89.

[18] See David Lewis. "Extrinsic Properties". In: *Philosophical Studies* 44.2 (1983), pp. 197–200, p. 1.

[19] Lewis, *On the Plurality of Worlds*, p. 86.

[20] See ibid., p. 3.

[21] See ibid., p. 95.

in non-modal terms."[22] Lewis agrees to the fact that his theory might not be fundamentally better than any other, but he proposes that his reduction of modality may outweigh the ontological cost of accepting possible worlds as real things.[23] The reduction of modality and therefore the rejection of a more modalist view of accepting modality as primitive comes due to the fact that by accepting the so called possible worlds, which he also just calls *worlds*, as there are no *impossible worlds* for him, we get modality for free.[24]

But I have not mentioned the most persuasive argument for modal realism, in my opinion. Because the fact that our world is as it is, with all its localised particles and laws of physical nature, and that I am myself, must be a contingent matter. Why are the physical constants how they are? One may argue for the case of there being an undiscovered explanation, just like quarks can explain nuclei, but then one must come up with an explanation for this new explanation, so one would fall down a hole of infinite regress. So, I believe just like Lewis that the way things are is not necessary, unless I were to accept the categorical hypothesis, which denies modality. But how to explain the facts that the world is how it is and that I am myself? Or to reformulate in terms of worlds: How can one explain that I am not living in a different world? The answer is that it is just arbitrary. This is because actuality is a matter of perspective. Just like I say about my world that it is actual, a possible individual will call their own world actual. Thus, our world is just one of infinitely many with no world really being special. So, it becomes just a contingent and arbitrary matter in which world I live and who I, the individual constituting being in my own world, actually am. And when rejecting modal realism, there seems to be no equally elegant way of explanation.[25] [26]

[22] Louis deRosset. "On the Plurality of Worlds: David Lewis". In: 2011, p. 4.

[23] See Lewis, *On the Plurality of Worlds*, p. 4.

[24] See Phillip Bricker. "David Lewis: On the Plurality of Worlds". In: *Central Works of Philosophy*. Vol. 5. Acumen Publishing, 2006, pp. 246–267, p. 6.

[25] See Lewis, *On the Plurality of Worlds*, pp. 129-131.

[26] If one considers quantum mechanics and believes in the many worlds interpretation, which might be considered closer to the mathematical formulation of quantum physics than the Born interpretation, you may even have one more reason to believe in modal realism. But this argument would go too deep to discuss it here in full.

3.2 Lewis's Critics

Now we will consider some important objections that were often made and are still made against modal realism. And we will see if they may really hold. Later, we will touch on the related theories of ersatzism and see whether they are more tenable.

3.2.1 Knowledge about Worlds

One point of criticism that also Lewis himself points out is the fact that we could never experience the other worlds and therefore, we should not be able to know about them.[27] And that is of course true in Lewis's mind, because how would you observe something that has neither a spatiotemporal nor a causal relation to you? But he argues that we know about worlds just like we know about mathematical objects, which also do not have a spatiotemporal or a causal relation to us.[28] In some sense just like mathematical objects are necessary in their existence, if you presuppose Platonism, so are Lewis's worlds. And how do we know about them? On the one hand, I already said that the contingent facts about our physical world can be explained by there being all other possibilities of ways things could have been without any logical contradiction. Additionally Lewis argues that, when we collect information about our world, we actually come closer of knowing which world is actually ours.[29] So in fact Lewis argues by just applying his modal concepts to epistemic modality, so we might know *a priori* that we live in any possible world, but we always have numerous worlds we can see as epistemically possible.

One may point out now that an important difference between worlds and mathematical objects is that "[m]athematical entities are abstract; Lewis's worlds are concrete"[30]. But I think this is somewhat misguided, because not only does Lewis step down on concreteness when saying that it he only says it "with some doubts"[31] that worlds are concrete, and I also say that mathematical objects can be seen as concrete in some sense too. If, for example, I have a cube defined precisely by its eight corner points in the three-dimensional

[27] deRosset, "On the Plurality of Worlds: David Lewis", p. 7.

[28] See Bricker, "David Lewis: On the Plurality of Worlds", p. 16.

[29] See Lewis, *On the Plurality of Worlds*, p. 112.

[30] Bricker, "David Lewis: On the Plurality of Worlds", p. 16.

[31] Lewis, *On the Plurality of Worlds*, p. 86.

vector space, it is one fully defined example object, with its abstraction, that is the cube in general. In some ways that makes this particular cube a concrete object. The problem with this argument may be that it makes mathematical objects even more problematic in modal realism because they exist necessarily, but they cannot really exist in a world, because they are not situated in space or time. So, viewing them as abstract may help arguing that they are indeed necessarily existing objects.

3.2.2 The Empty World

Lewis's worlds are seen as mereological sums of their objects. *Mereology* is the theory of parthood and defines a certain *being part of*-relation that follow certain criteria. They are: "Everything is part of itself. Any part of any part of a thing is itself part of that thing. Two distinct things cannot be part of each other."[32] The problem this conception runs into is that there are no empty parts, like we would find in set theory with the empty set. Therefore, there can also be no empty world, which some argue to be a desired trait of modality, saying that the whole actual existence of a world is contingent.[33] Lewis argues instead that there is no need for an empty world, because a world with exactly nothing in it would be nothing more than that: nothing, not even a world. The world is only constituted by its things, even a single point of spacetime could constitute a non-empty world.[34] But I feel like he does not really answer the question completely, because his argument is only supported by his thesis of plural worlds. But I can also see reasons why one should not think that it would be possible for there to exist nothing. Firstly, even when our imagination of worlds may never complete the space of worlds, like with alien properties, it is very hard to imagine a world where there is nothing at all. And secondly, even without an empty world we are able to say that every object and individual exists contingently, because it is a different thing to say that it is possible that nothing exists than to say that it might be for every actual thing that it might not exist. Still it seems also to be a contingent matter that there exists anything at all.

[32] Achille Varzi. "Mereology". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Spring 2019. Metaphysics Research Lab, Stanford University, 2019.

[33] See Melia, *Modality*, p. 112.

[34] See Lewis, *On the Plurality of Worlds*, p. 73.

3.2.3 The Principle of Recombination

There have been many objections made in regard to Lewis's principle of recombination; because, if we were to take all the worlds there are and combine them under this principle to a new world, we would end up with a bigger world than our biggest world from the worlds we assumed as being complete. So, Lewis counters this argument, which was put forward by Forrest and Armstrong in 1984, by saying that there should be a natural limit of the shape and size of worlds, which he argues is better than none. Therefore Lewis's worlds are constructed by the principle of recombination restricted by the size of the world.[35] The problem still does not seem to be satisfyingly solved, further we might consider infinitely (countable, uncountable and bigger) many worlds and infinitely big spacetime.

Another weakness of the principle of recombination is that it may be seen as too weak to serve modal realism. As Louis deRosset criticises, the principle of recombination is not able to produce alien properties, it does not *secure fit*, which means that as a reduction it cannot explain main features of modal space. Furthermore it is argued that the restriction part of the principle is actually smuggling primitive modality into the theory, because "as long as there is enough space in that world" should be read more primitively as "as long as it is possible to be things of those sizes and shapes, and in that arrangement."[36]

3.3 Linguistic ersatzism

Related to modal realism are the so called ersatzisms, Ersatz modal realisms or moderate modal realisms, which Lewis probably does not oppose as much as other modal theses, otherwise he would not have given them a whole chapter to explain them and talk about them. Even though, at the end of it, he gives reasons to reject them. All ersatzisms propose only one existing world with countless abstract *ways*, which may be called *ersatz worlds*. The most promising theory of the ersatzisms for Lewis, it seems, and also in my opinion is the linguistic ersatzism or also called book realism, which views its ersatz worlds

[35] See Lewis, *On the Plurality of Worlds*, p. 91 & pp. 102-103.

[36] See deRosset, "On the Plurality of Worlds: David Lewis", pp. 9-11.

as maximal consistent sets of sentences.[37] Even though some book realists renounce the maximality of these sets and consider instead *partial stories*.[38]

The set up for this approach leads already to some questions one should ask. The most important one is: How to define our language so that it fulfils its purpose of giving us an unambiguous description that does not rely on cultural context? One may define such a sentence as a "sequence of phrases which are its immediate constituents, let each phrase be the sequence of phrases which are *its* immediate constituents, and so on down to phrases which are single words."[39] So, just like we would define logical formulas and sentences by building up from single symbols like connectives, quantifiers, predicate symbols and so on, which we could call words, we would in a similar way build our sentences here. The only big difference here is that words must be in some way also defined, unlike in pure logic, where we are able to just work with abstract properties without any upfront meaning. And the solution is of course to refer to instantiations in the real world. So, when I want to refer to objects in another world, like for example horses, the word *horses* gets its definition by referring to all the spatiotemporal instances of horses in our world. One must keep in mind that the atomic resources of such a *Lagadonian* language are very sparce in comparison to our logical *words*. Instead we rely on the fact that "each object has its own name, and each universal as its own predicate."[40] Thus, *words*, which connect the predicates and names, must come from the set-theoretic structures of our sentences.

Universals are of course very helpful here, because they provide us with some very primitive notion of properties, while also explaining how the same properties of different objects can even be the same. This is because, if a thing is the basis of a universal it is also composed of an instantiation. *Tropes*, as a counter theory to universals, behave more like individuals themselves, rejecting a primitive notion of properties but instead embracing duplication between objects of the same property as primitive.[41]

This gives us a very rich and expressive language to build sentences to describe *ways*

[37] See Lewis, *On the Plurality of Worlds*, pp.136 & 146.

[38] See Tony Roy. "In Defense of Linguistic Ersatzism". In: *Philosophical Studies: An International Journal for Philosophy in the Analytic Tradition* 80.3 (1995), pp. 217–242, pp. 221-225.

[39] Lewis, *On the Plurality of Worlds*, pp. 142-143.

[40] Roy, "In Defense of Linguistic Ersatzism", p. 219.

[41] See Lewis, *On the Plurality of Worlds*, p. 65.

the world could have been. There is even the advantage, Lewis admits, that this way of speaking about possible worlds is ontologically very unproblematic, in comparison to his view. While he needs to assume the existence of unobservable collections of objects, which exist for themselves spatiotemporally isolated, in linguistic ersatzism there is already everything there to make the ersatz worlds work. Even rejecting set theory may be compatible with thinking ersatzism as true.[42]

But even then, Lewis argues, the ersatzer is not able to describe modal worlds to their fullest extent. The first problem he points out is that of cardinality. And even though Lewis resolves the problem by accepting a more infinitary and less natural language, I want to discuss the issue here to save some natural properties of the language we might have in ersatzism. When spacetime is to be seen as continuous, which it often is in physical descriptions of the world, it can be argued that there is a continuum of infinite spacetime points, each of which must therefore be described by whether it is occupied and if yes by what it is occupied by. But there can only be a finite number of *words*, Lewis argues, which makes the set of sentences, which are a finite string of words, only countable infinite. Therefore it is impossible to describe all possible worlds with linguistic ersatzism.[43] But in my opinion, the argument that there are an uncountable amount of possible worlds does not hold automatically. Lewis is very much mislead by his physicalist view, I think, because spacetime is *often described* by using the real number line. But that does not mean that spacetime *is actually* behaving like $\mathbb{R}^4$. In fact, considering the atomistic (or one may say in a more modern way *quantised*) nature of quantum mechanics, it becomes more and more clear that there may not be a continuum of spacetime, just like there is no continuum of the number of decaying uranium atoms over time, which is anyway often described through a real exponential function.

Lewis's physicalism in general seems very obscure to me because it presupposes that the way we experience our world in its physicality is the way the world is. But even scientific theories do not describe the world as it is, they are only models designed through the principle of "which hypothesis fits better can stay, the other must be discarded". Just like Newton's mechanics could not stand the test of time, because it was at some point un-

[42] See Lewis, *On the Plurality of Worlds*, p. 143.

[43] See ibid., p. 143.

able to explain certain behaviours in nature, so will the mechanics of Einstein be refuted, as soon as there is a better theory explaining the strange phenomena, we are currently unable to explain. So, we actually have no idea how the objective world looks like and we never will, because we cannot see beyond our subjective view and our senses and we must always build models, which may fit better to the world, but they will never become the world. But then: What are we actually talking about, when we talk about the world? Maybe we should understand the term *world* not as *the* world, but more as one's own world. An object being present in the observer's present time and space seems to have an important role in the sense that talking about things only makes sense from this perspective, because this is the only situation where talking happens: An individual talking about the things and their properties in the world of this individual. So, one could reformulate the criterion for *being a worldmate* to that of "all things that have a relation of experience to me, are in one world", which sounds then a lot more like what ersatzism tells us.

And for all the same reasons as stated before, it is not sensible to me to talk about indiscernible worlds. And while for Lewis there is not such a big deal with the Ersatzist being unable to discriminate indiscernible worlds, he does say that the language is still too weak.

> *The problem of indiscernible individuals, on the other hand, is serious. Certainly, it is at least possible that there should be many indiscernible individuals – alike in their intrinsic natures, and their extrinsic properties as well. [...] But we do not have correspondingly many indiscernible ersatz possible individuals, all actualised according to this ersatz world. One must do for all.*[44]

But Tony Roy, for example, blocks this argument by saying that "our ability to distinguish indiscernible objects seems as good as our ability to name them."[45] His idea is that ersatz worlds and individuals are not merely sets of sentences but should be seen more as a story one tells, where one is clearly able to distinguish two indiscernible objects through names. Furthermore Roy proposes the idea of arbitrary names for non-actual individuals as placeholders,[46] which I think is philosophically totally unproblematic, because names

[44] See Lewis, *On the Plurality of Worlds*, p. 149.

[45] Roy, "In Defense of Linguistic Ersatzism", p. 227.

[46] See ibid., p. 228.

themselves are no more than arbitrary assignments of words to objects.

The other weakness often pointed out about Lagadonian language is the inability to work with alien properties. As already mentioned, Lewis can presuppose the existence of alien properties in worlds, because they contain anything that is possible. In ersatz modal realism however, there are no predicates for alien properties. Instead, these fundamental properties may be described by the fact that they are not identical to any other property there is in the actual concrete world. So the real problem arises because the modal realist is able to add and permute all the different alien properties throughout the alien worlds, while the modal ersatzer would have to conflate different worlds for there to be different alien properties at once, or so the argument goes.[47] At first this line of argumentation seems plausible, but it leaves out the possibility that firstly we may just define a second property by giving it another arbitrary name and secondly we are also able, and I would even say *inclined*, to give these properties certain behaviours. Lewis gives the example of a philosopher in a world, where there are no quarks, and he argues that the philosopher could never know about the properties of quarks, like the flavours and colours of quarks. So these would be alien to him.[48] But why should someone not be able to at least build a mathematical model to describe such properties with this behaviour? Of course, it would be a stupid model in that world because it would suppose more things than necessary without giving more explanatory power. But it is nonetheless possible and therefore it should be possible for us to describe all alien properties there could be.

Another important problem Lewis sees in ersatzism is that it needs to take modality as primitive, while Lewis's modality emerges plainly from the existence of his worlds, or so it seems. The first reason for an ersatzist to need to appeal to modality, is that the concept of maximal consistency tells us that it is not *possible* to add or remove a sentence from our world-story. The second reason Lewis gives is that there may be implicit representations of facts in our set of sentences, because "there might be a single sentence which implies so-and-so but doesn't just mean so-and-so because it implies more besides; or there might be a finite or infinite set of sentences which jointly imply that so-and-so."[49]

[47] See Bricker, "David Lewis: On the Plurality of Worlds", p. 24.

[48] See Lewis, *On the Plurality of Worlds*, p. 159.

[49] See ibid., p. 151.

But what Lewis describes here could not be the case for a maximally consistent set of sentences, because then it would not have been maximally consistent, unless we have propositions that we cannot formulate by our language. But I think it is not plausible to think that we could find such sentences. Therefore, this point refers also only to the fact that defining maximal consistency needs modality. This, Lewis admits, is not an argument for abandoning linguistic ersatzism but he sees it, like many other aspects, as an advantage in favour of his own theory.[50] Because, why should it be bad to have modality assumed as primitive? Roy asks this as well and he argues that it is actually no problem at all, because all theories who concern themselves with modality can never say why their objects of modality, be it possible worlds or ersatz worlds, determine modality. So, the ersatzer would respond to Lewis's claim with the question of why his worlds make modality happen, in the same way he asks what makes this or that set of sentences a world-story.[51] But still, there is much argument to be made against an axiomatisation to decide which sentences can be part of one world-story or not because it might give away some of its explanatory power.

In conclusion, it seems to me that there is a great deal in linguistic ersatzism or book realism over genuine realism. While modal realism is indeed a very powerful tool, giving us all we need for any good modal statement from the beginning, it does have its ontological extravagance in assuming all those worlds. And while linguistic ersatzism is on many levels on par with modal realism – it only needs language and the one world, which we all generally agree exists – it does need to construct its possible worlds in a careful way. And language as a tool to even talk about worlds and modality is just by its nature quite weak and incomplete in its descriptive power. We never do and need to describe a world by its fullest to be able to talk about it, which makes partial stories a very attractive concept.

4 Transworld Identity

In the first chapter we already talked about domains of worlds that share their individuals in one way or another. So, in that sense we already considered the technicalities of transworld

[50] See Lewis, *On the Plurality of Worlds*, p. 156.

[51] See Roy, "In Defense of Linguistic Ersatzism", pp. 236-237.

identity, that is "the notion that the same object exists in more than one possible world."[52] But is it even possible that we could have identity across possible worlds? Can we allow an object to be in different worlds, where it may have differing properties? While we will generally talk about transworld identity and its problems in terms of Lewis's ideas, this conception can be applied to all kinds of realisms, which assume the existence of worlds at least conceptually as true, not only genuine realism.

4.1 Haecceitism and Essentials

Believing in transworld identity could have unwanted consequences, some of which we will discuss; but, there are some people who argue that this problem of transworld identity is actually no problem at all, because there is either no coherent formulation of the problem or the problem, if formulated in a coherent way, will solve more or less itself.[53] Such a believe is often based upon or can only be uphold by the belief in *essential properties* or other representations, which are "responsible for a substance's individuation and identity"[54]. The belief in such a de re individuation without any qualitative individuation, like essential properties, is called *haecceitism*.[55] One has to keep in mind that haecceities and essentials do not exclude one another, instead they may be regarded as extreme versions ranging from *de re* representation of properties via representation by some or more haecceitistic differences up to the point where there are only haecceitistic differences between individuals.[56]

One problem that could occur if we deny essential properties or haecceities of individuals, is Chisholm's paradox,[57] which might make transworld identity problematic in the first place. We could have for example two persons, A and B, in a world Γ; they have, of

[52] Mackie and Jago, "Transworld Identity".

[53] See Peter van Inwagen. "Plantinga on Trans-World Identity". In: ed. by Peter Tomberlin James E.and van Inwagen. Springer Netherlands, 1985, pp. 101–120, p.101.

[54] Richard Cross. "Medieval Theories of Haecceity". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Spring 2022. Metaphysics Research Lab, Stanford University, 2022.

[55] See Lewis, *On the Plurality of Worlds*, p. 221.

[56] See ibid., p. 239.

[57] See Roderick M. Chisholm. "Identity through Possible Worlds: Some Questions". In: *Noûs* 1.1 (1967), pp. 1–8, p. 1-5.

course, different properties in this world. But now, consider that there must be an accessible world from Γ, where some of their properties are swapped or just slightly changed. And if we always play this game from the next world, again and again, we will eventually arrive at a world Ω, which is exactly like Γ, except that the one individual we called there A is played by B, because A has in Γ the exact same properties as B in Ω. This is of course only possible because we do not have essentials or haecceities which must be preserved in these individuals. The problem we run into now is that Γ and Ω are indiscernible, one could even say they are identical in every way,[58] so we should conclude that A in Γ is really identical to B in Ω. But because identity is not only symmetric but transitive, we then must conclude that $A = B$ no matter in what world they are in. And because we could choose any two individuals or objects, this is very problematic. So, the solution for this is either to abandon transworld identity in full or we have to admit that everything in every world must have essential properties or haecceities, as they prohibit such a switch of identity.

One very simple argument for transworld identity is to appeal to common sense, the way we talk in everyday language and even the way one should understand modalitiy *de re*. When we say, "Socrates could have been a carpenter", we mean of course to refer to him and not any someone else. We will see how Lewis, who believes in fully disjunct worlds, solves this problem, by stating modalities *de re* through counterpart relation. But an even more convincing argument, supporting some sort of haecceitism, is the argument of identification. Whereas one urges the question, if there exists an object c in this world Γ, how to *identify* this object in another world Δ. How is it knowable which thing from all the things existing in Δ corresponds to c? There is indeed no clear and strict answer to this question without assuming transworld identity in the first place. Some philosophers therefore may advocate for a criterion, which makes it possible to identify a certain thing. That means, that for the identification of an actually existing person in another world "there must at any rate be *some property or other* that he and he only has"[59]. But even that may not be convincing for some, because all this talk about other worlds, their differences

[58] If we were to accept haecceitism these worlds may be indiscernible and physically identical, but there would be the haecceitistic difference between our two individuals.

[59] Alvin Plantinga. *The Nature of Necessity*. Oxford, England: Clarendon Press, 1974, pp. 96-97.

and properties paints a picture of us looking into other worlds, which is merely never the case, no matter if you are a genuine or an ersatz modalist. Another problem arises when we only identifying something just through its name. Plantinga argued that *proper* names do express essence,[60] therefore one has to believe in essentials when wanting to take names and their meaning seriously. Now the problem arises for the denier of transworld identity that, because a thing only exists in one world, all its properties are essential,[61] which makes the concept of essentials a bit meaningless. So, it seems that much about the question of transworld identity comes down to the question of whether essentialism is true. We cannot have both.

Lewis's opinion about essential properties and haecceities is that they do not exist. One has to keep in mind that essential properties are to be separated from necessary properties, like triangularity of triangles or bodies being extended in space. Therefore, he is only against the concept of a *thisness of objects* constituted by there being contingent properties that prevail throughout the worlds. Instead, he argues that possibilities must be differentiated from possible worlds, because "possible worlds are some of the possibilities [but] *any* possible individual is a possibility, and not all possible individuals are possible worlds."[62] He therefore concludes that there should not be other criteria to discriminate qualitatively indiscernible worlds (i. e. haecceities), but there are not enough possible worlds to produce all possibilities. So, instead of talking about haecceitistic differences between worlds he favours to talk about "differences between worldmate individuals"[63]. The idea here seems to be that one only compares the possibilities of an individual, which one may see as some kind of *miniworld*, and therefore dodging the problem by viewing *de re* modality in a way as some *de dicto* modality cut down to individuals. The advantage, of course, is that speaking about *de re* modality is best done by talking about individuals and not by talking about differences between worlds.

[60] See Plantinga, *The Nature of Necessity*, pp. 77-81.

[61] See ibid., p. 98.

[62] Lewis, *On the Plurality of Worlds*, p. 230.

[63] Ibid., p. 235.

4.2 Counterpart Theory

As already discussed in the last chapter, Lewis's idea of worlds is that they are spatiotemporally and causally isolated sums of all things there are. This gave us already the conclusion that they must all be totally disjunct, so there is no thing existing in two worlds. Therefore, Lewis is in every way inclined to argue against transworld identity, which brings us to his alternative: *counterpart theory*, which is applicable to all kinds of realisms.

The counterpart relation C^2 can be seen as an alternative to the two-place predicate of identity $=^2$. The huge difference here is that C^2 is very unrestricted while $=^2$ must always be an equivalence relation, which is one reason we got Chisholm's paradox. Still, there are some conditions put forward to the counterpart relation, which Lewis outlined as follows as part of his postulates constituting counterpart theory.

> *P3: Whatever is a counterpart is in a world.*
>
> *P4: Whatever has a counterpart is in a world.*
>
> *P5: Nothing is a counterpart of anything else in its world.*
>
> *P6: Anything in a world is a counterpart of itself.*[64]

The first three are not surprising. They tell us that counterparts must reside in worlds, because where else should they be? And the counterpart relation is always between worlds, when it is between different individuals, also not so surprising. But the last one of these postulates is a bit more interesting, I think. While it does seem a bit unnecessary to have a counterpart relation not from one world to another, when we are interested in an alternative for transworld identity, it implies that everything may possibly play its own role, which is something most people would agree to.

There is no further thorough definition of the counterpart relation, because *the* counterpart relation does not exist. Lewis bases these on a notion of similarity, where it depends on how one weighs different kinds of similarity and therefore different counterpart relations.[65] Therefore counterparts as a replacement for transworld identity do not only avoid

[64] See David Lewis. "Counterpart Theory and Quantified Modal Logic". In: *The Journal of Philosophy* 65 (1968), pp. 113–126, p. 2.

[65] See ibid., p. 115 & See Lewis, *On the Plurality of Worlds*, p. 255

Chisholm's paradox, but offer a variety of lenses to look upon modality, like looking for similarity of origin to give us something that behaves like origin essentialism[66] or looking for no similarity at all giving us logical space.

4.3 World Dependent Properties and Leibniz's Law

A strong argument to be made against counterpart theory is some analogy of time in respect to possible worlds. Like I already pointed out in the first chapter, modal statements *de re* require us to give our objects not only a property, but a property assigned to a world. The same happens when talking about the same person or thing over time. At one point Socrates was a young boy, where he was only a meter tall, later he grew older. This could be a seemingly uncontroversial point to start an argument against Lewis, because when we always speak of the same Socrates with different properties over time, we should be able to say the same about Socrates over different worlds, where he has also different properties.[67] But is that really so? Is the old and wise Socrates really the same person as the young boy he was many years before then? I think that, while the analogy of time is very fitting, because we also should be able to use worlds for temporal interpretations, this argument runs into the same misconception as other arguments for trans-world identity. While the young and old Socrates do have very much in common, which is their past and their origin, this is nearly all that connects them, so they are not the same person.

The argument here made is one that uses Leibniz's law, which states that everything that is identical to something else must share all the properties of its identical.[68] Socrates cannot be old and young at the same time. But this formulation sounds like Socrates may have contradicting properties if we assign them to different times or worlds. So, the real question is if we want to understand real properties, as understand in Leibniz's law of indiscernibles, as properties assigned to a world or if we want to hold on to unextended properties. This gives us two general positions on the existence of objects throughout worlds, or to stay in our analogy, throughout time, which Lewis also discusses.

[66] See Sam Cowling. "Haecceitism". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Fall 2016. Metaphysics Research Lab, Stanford University, 2016.

[67] See Mackie and Jago, "Transworld Identity".

[68] See Noonan and Curtis, "Identity".

Objects might have *perdurance* or *endurance*. An object endures through time if it is wholly present at more than one time. And it perdures if its whole present is imbedded in one moment. These two ideas correspond exactly with the ideas of transworld identity and counterpart theory.[69] Lewis argues against endurance by talking about the question of how change in objects are to be explained, especially concerning shape, which is an example of a temporary intrinsic. Accepting endurance would conclude that there cannot be any such intrinsic properties, because they belong by the nature of intrinsic properties always to that thing. So, shape would be seen as a relation "that something with an absolutely unchanging intrinsic nature bears to different times."[70] This is unacceptable to Lewis, also because shape is no relation.

The present world, the observer or the present time in our analogy play quite important roles, when modal sentences are said. When I say "The sky is cloudy", what I actually mean is that now, in the time I say this, the sky, which is located above myself, is cloudy (If you are a modal realist, you could add that you mean all this and in the actual world). But no one would add these unnecessary details about the actual here and now. In the same sense the meaning of the sentences "It rained eight hours ago" and "The house is a kilometre away" will drastically change depending on where and when you are. It seems that there is a big importance about this kind of present (being here, now and in the actual world) allowing us to talk solely with properties without any reference to a certain world. Therefore, I think we should be able to attribute properties to things without any reference of the world there are in generally, therefore endurance and transworld identity should be rejected and instead one should talk about counterparts and disjunct worlds, where one does not need to specify in which world an object is.

5 Conclusion

This discussion of modal realism and transworld identity is far from exhausted. But I hoped to show the fundamental ideas and problems they entail. While modal realism has its advantages in assuming its worlds, it is too ontologically extravagant to accept it;

[69] See Lewis, *On the Plurality of Worlds*, p. 202.

[70] Ibid., p. 204.

especially compared to modal ersatzism, which is closer to how modality is used, in my opinion. And while transworld identity seems like an intuitive feat of modality, it seems more like an abbreviation of certain counterpart theories which assume a certain similarity which one might like to call essence.

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